O.H: None tomorrow

Instead today 3PM–7PM (PST)

I. Lagrange multipliers

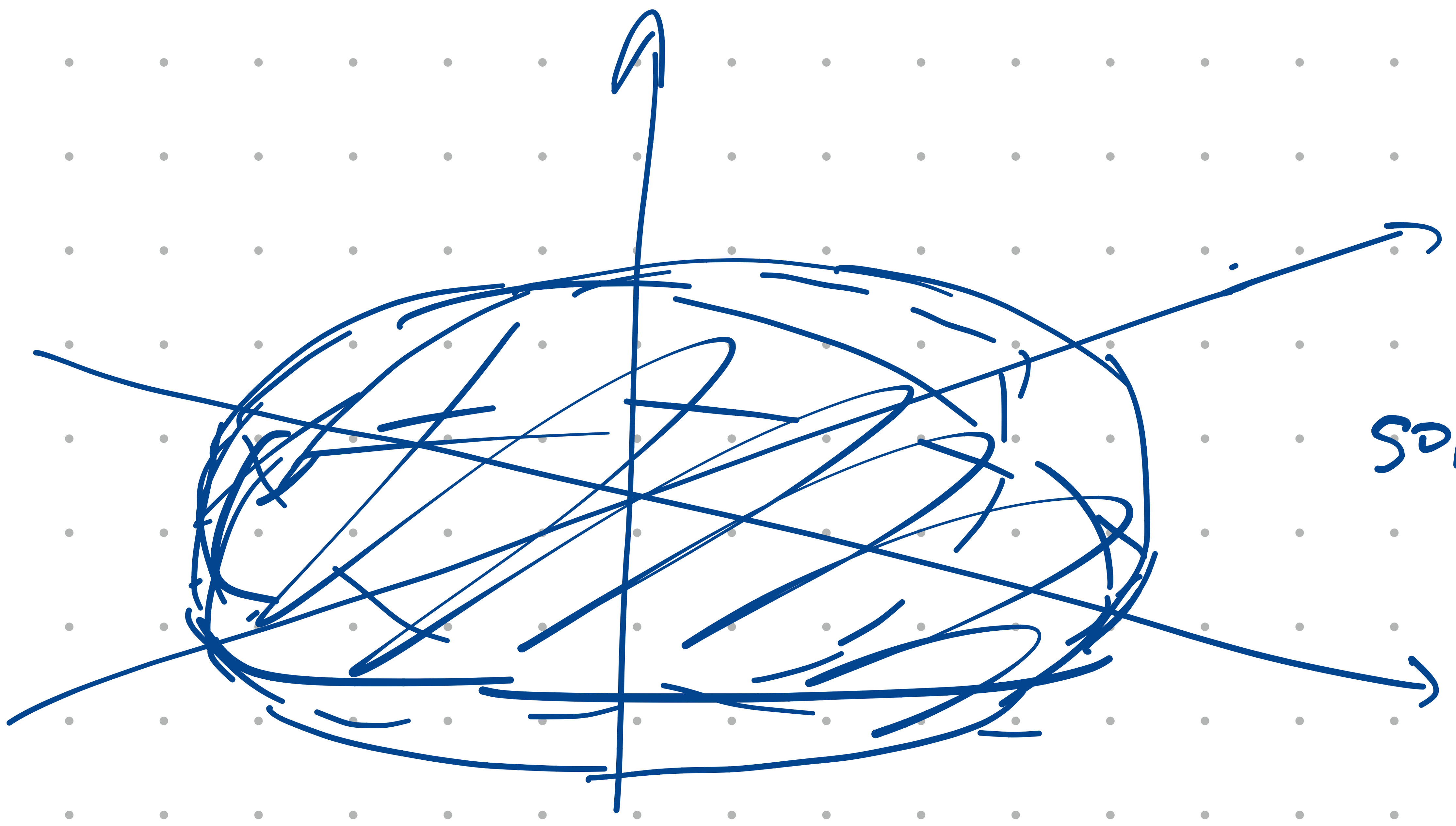
II. Making use of "conservative"; interpreting $\int_C \vec{F}\cdot d\vec{r}$ intuitively

Quiz 9 #2

III. 15.9 change of var.

I. Find abs. max & min of $f(x,y,z) = 2x^2 + y^2 + z^2$ on

$$x^2 + y^2 + 2z^2 \leq 1$$



solid ellipsoid

(0. Convince yourself that the abs. max and/or min that you're seeking actually exist. e.g. EVT)

1. Decompose $x^2+y^2+2z^2 \le 1$

(A)

$x^2+y^2+2z^2 < 1$

(B)

$x^2+y^2+2z^2 = 1$

2. Find candidates (a finite list of possible locations where the abs max & min might be), subject to any equality constraints. Then check they satisfy any inequality constraints.

A: No equality constraints $\Longrightarrow$ $\nabla f = \vec{0}$

i.e. candidates = critical pts.

$$\langle 4x, 2y, 2z \rangle = \langle 0, 0, 0 \rangle$$

$\boxed{(0,0,0)}$ check: $0^2 + 0^2 + 2 \cdot 0^2 = 0 < 1$ ✓

B: Constraint $\underbrace{x^2 + y^2 + 2z^2 - 1}_{g(x,y,z)} = 0 \Longrightarrow \nabla f = \lambda \nabla g$ to find candidates.

$$\begin{cases} x^2 + y^2 + 2z^2 - 1 = 0 \\ 4x = \lambda 2x \\ 2y = \lambda 2y \\ 2z = \lambda 4z \end{cases}$$

⚠ $4x = \lambda 2x \;\not\Longrightarrow\; 4 = 2\lambda$

unless you assume $x \neq 0$.

So really: $4x = \lambda 2x \Longrightarrow$ $\boxed{\text{either } 4 = 2\lambda \text{ or } x = 0.}$

Don't forget about the $x=0$ case!

Case $4 = 2\lambda$: $\lambda = 2$

$y = 2y$, $z = 4z$

so $y = z = 0$.

$x^2 - 1 = 0 \Rightarrow x = \pm 1$

Candidates are

$$\boxed{(1,0,0),\ (-1,0,0)}$$

Case $x=0$: $y = \lambda y$ so either $\lambda = 1$ or $y = 0$

subcase $\lambda = 1$: $z = 2z$ so $z = 0$.

$0^2 + y^2 + 2 \cdot 0^2 - 1 = 0 \Rightarrow y = \pm 1$

$$\boxed{(0,1,0),\ (0,-1,0)}$$

subcase $y=0$: $\quad 0^2+0^2+2\cdot z^2-1=0$

so $z = \pm\frac{1}{\sqrt{2}}$

$$\boxed{\left(0,0,\frac{1}{\sqrt{2}}\right),\ \left(0,0,-\frac{1}{\sqrt{2}}\right)}$$

3. Compare the values of f:

$f(0,0,0)$

$f(1,0,0)$

$f(-1,0,0)$

$f(0,1,0)$

⋮

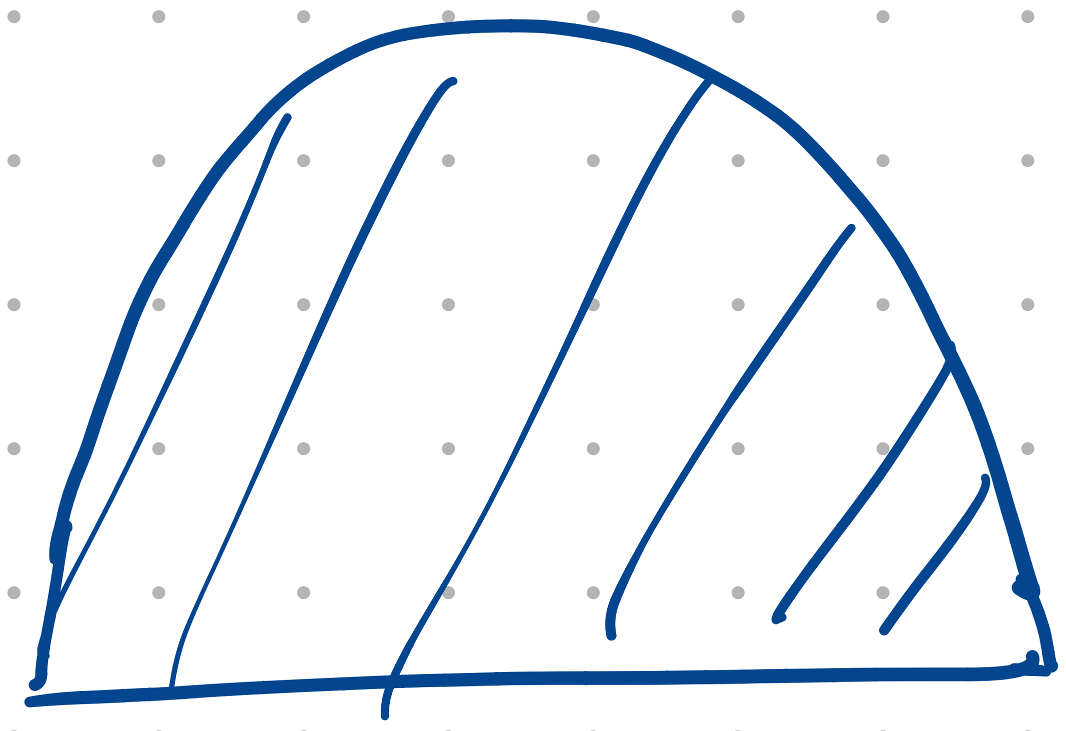
etc.

biggest is abs max
smallest is abs min

Q: Where does the 2nd deriv test fit into this?

A: it doesn't. It's for instead analyzing local behavior of a function $f(x,y)$ when no equality constraints are present.

Rmk.



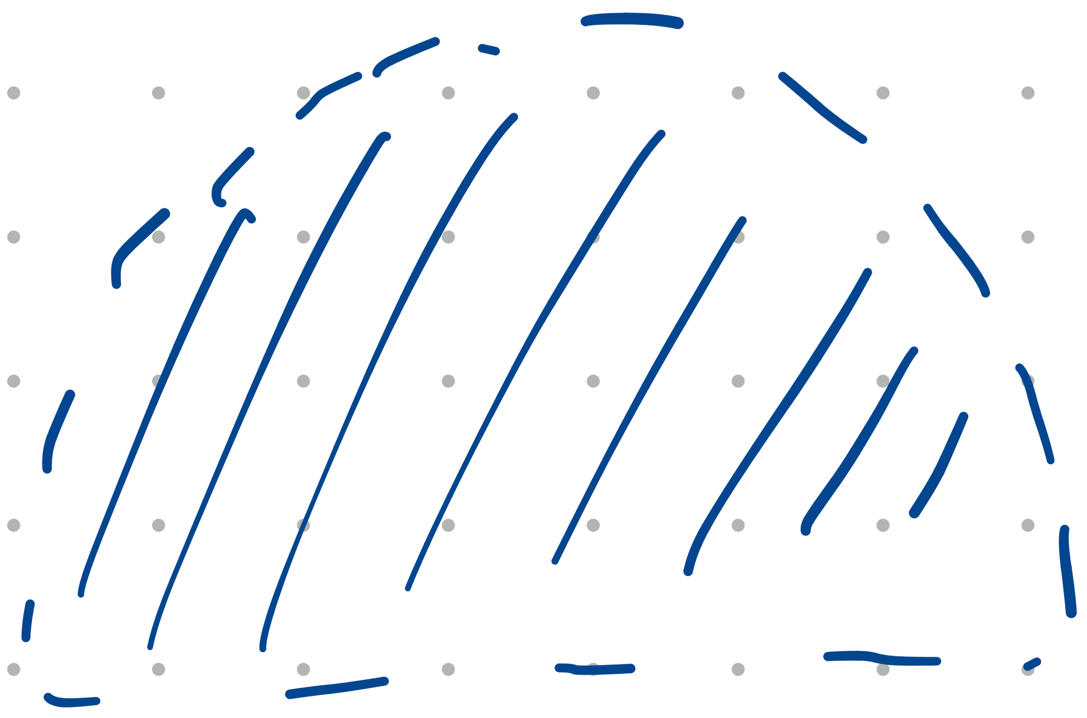
$y \leq 1 - x^2$

$y \geq 0$

$f(x,y)$ some fn...

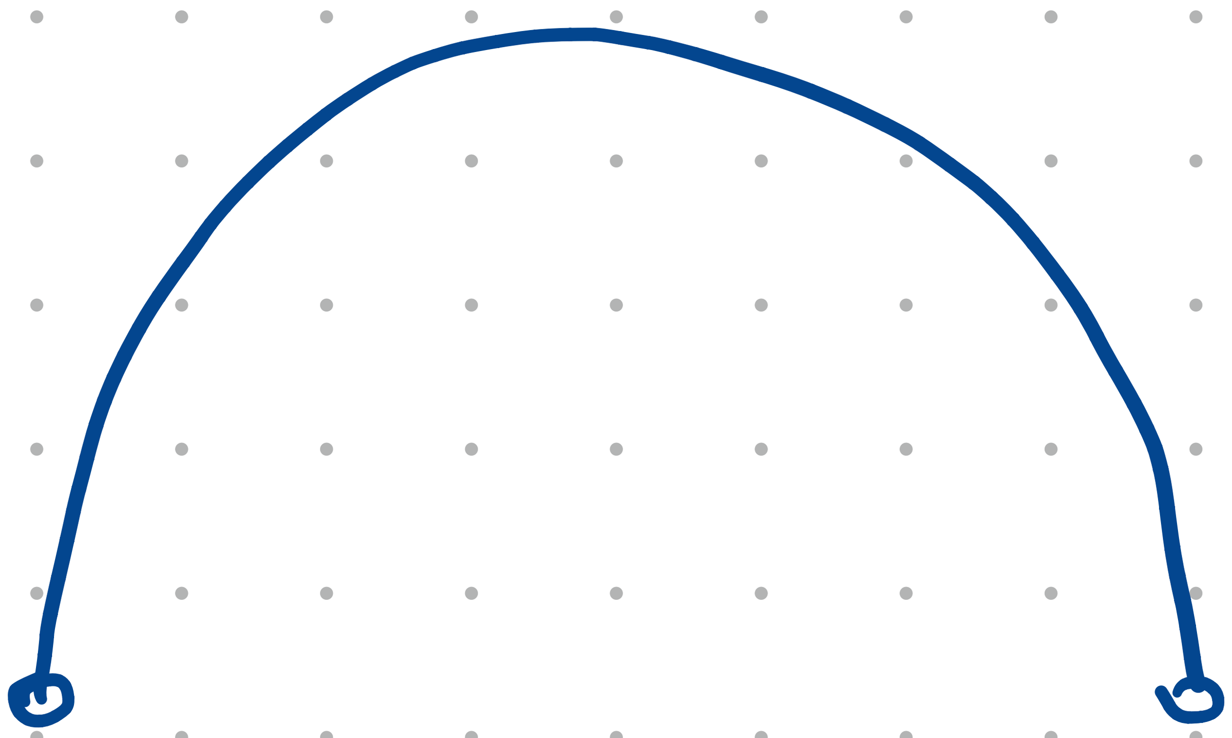
decomposes as:

$=$



$y < 1 - x^2$

$y > 0$

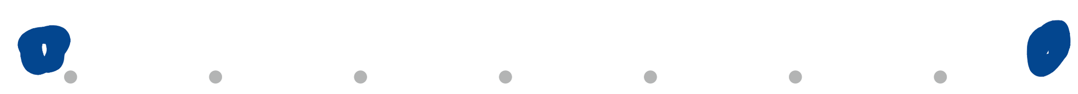


$y = 1 - x^2$

$y > 0$

$y < 1 - x^2$

$y = 0$



$y = 1 - x^2$

$y = 0$

II.

1.1: Just directly apply FTLI

1.2: (I meant $f \cdot \nabla f$

vector field.

scalar multiplication

scalar field

If you misinterpreted as fn composition,

you would've gotten "not enough info")

Either recall $\nabla(f^2) = 2f\nabla f$

or directly compute $f\nabla f$ since I gave you f.

2 a) Write $\vec{F} = \langle P, Q \rangle$, then

$$Q_x - P_y = (x + 5e^y) - (x + 5e^y) = 0.$$

Moreover, the domain of $\vec{F}$ is $\mathbb{R}^2$, which is simply connected, so this criterion $Q_x - P_y = 0$ is sufficient to conclude $\vec{F}$ is conservative.

b) A lot of ppl set up the integral by parametrization:

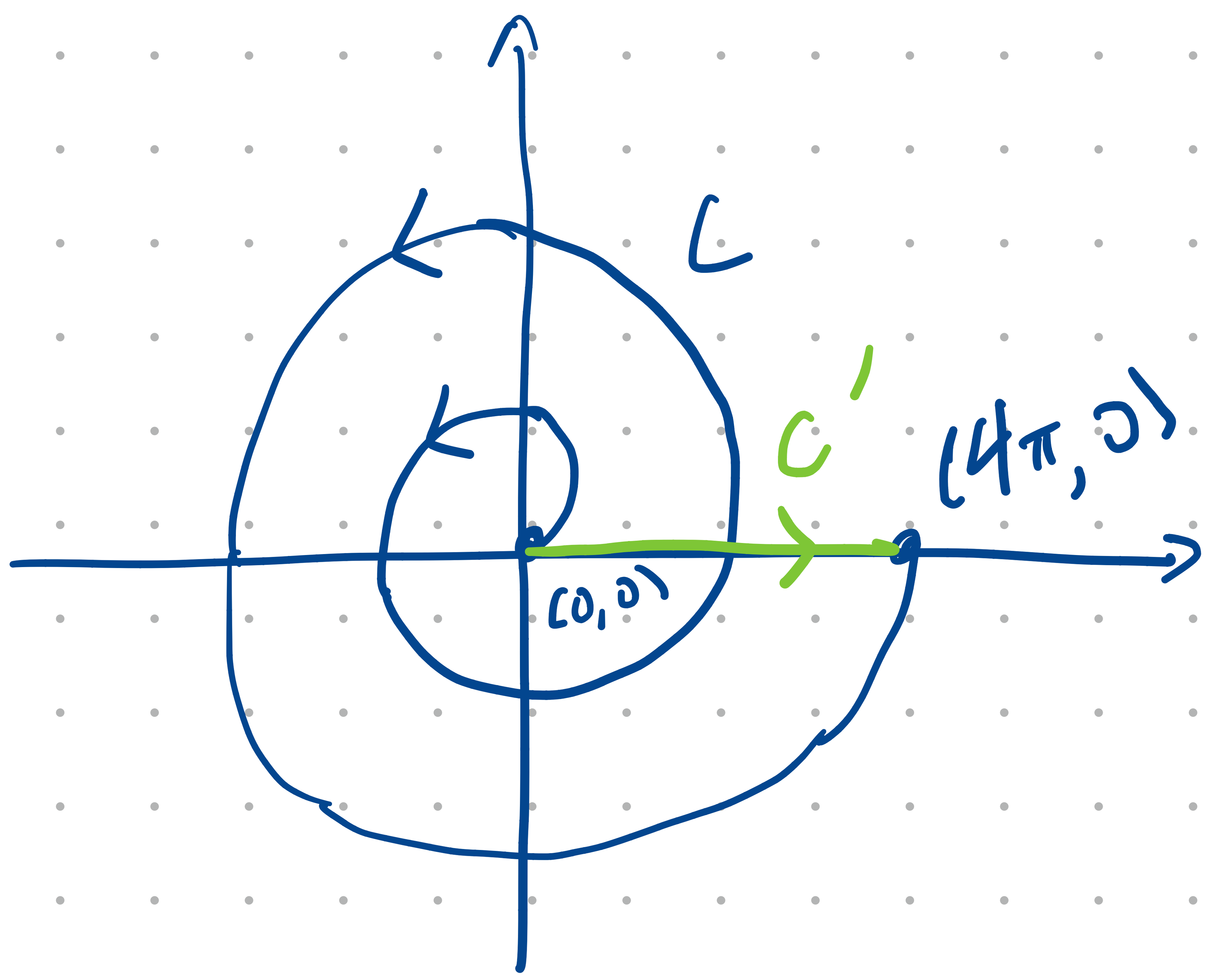
$$x = r\cos\theta = \theta\cos\theta$$

$$y = r\sin\theta = \theta\sin\theta$$

$$\int_0^{4\pi} \langle \theta^2 \sin\theta\cos\theta + 5e^{\theta\sin\theta}, \cdots \rangle \cdot \langle \cos\theta - \theta\sin\theta, \cdots \rangle \, d\theta$$

Looks painful!

Not taking advantage of what we learned from (a).



One interp. of (a) is that $\vec{F}$ is path-independent

so

$$\int_C \vec{F} \cdot d\vec{r} = \int_{C'} \vec{F} \cdot d\vec{r}$$

for any curve C' that starts @ $(0, 0)$ and ends @ $(4\pi, 0)$

C': $\vec{r}(t) = \langle t, 0 \rangle$
$0 \le t \le 4\pi$

$$\int_{C'} \vec{F} \cdot d\vec{r} = \int_0^{4\pi} \langle t \cdot 0 + 5e^0, \text{mm} \rangle \cdot \langle 1, 0 \rangle \, dt$$

$$= \int_0^{4\pi} 5 \, dt = \boxed{20\pi}$$

Another take: from (a) we know $\vec{F}$ has a potential fn f, i.e. $\nabla f = \vec{F}$. Let's try and find f, and use FTLI.

$$f_x(x,y) = xy + 5e^y$$

$$f(x,y) = \frac{1}{2}x^2y + 5xe^y + C(y)$$

$$\sin(y^2) + \frac{x^2}{2} + 5xe^y = f_y(x,y) = \frac{1}{2}x^2 + 5xe^y + C'(y)$$

Let $C(y)$ be some antiderivative of $\sin(y^2)$.

(Every continuous single-var fn has an antideriv!)

Then $f(x,y) = \frac{1}{2}x^2y + 5xe^y + C(y)$ is a pot. fn. for $\vec{F}$.

$$\int_C \vec{F}\cdot d\vec{r} \underset{\text{FTLI}}{=} f(4\pi,0) - f(0,0)$$

$$= (0 + 5\cdot 4\pi\cdot e^0 + \cancel{C(0)})$$

$$- (0 + 0 + \cancel{C(0)}) = \boxed{20\pi}$$